

The Conditional Value at Risk as a Risk Constraint in Optimal investment portfolio: An Analytical Study

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Abstract: *This study hopes for measuring the value at risk (VaR) of a group of investment portfolios and using it as a constraint in portfolio composition. This was come true utilizing the conditional value at risk (CVaR), also known as the mean value at risk (Mean-VaR), which is a proxy for risk in investment portfolios. The study resorted to returns for the period (2012-2024) and employed SPSS statistical software. For accomplishing its objective, this study adopted the main hypothesis: "The conditional value at risk is a reliable measure of risk and therefore giving an account for a constraint on the risk of the optimal investment portfolio."The study fulfilled several conclusions, the most significant of which is that the conditional value at risk speaks for a financial constraint on the risk of the optimal investment portfolios for the study's periods. The most prominent recommendations coming out from the study are that investment specialists, particularly investment portfolio managers, should embrace techniques and models for calculating and estimating the various aspects of risk involved in investment decision-making, including the investment portfolio.*

Keywords: investment portfolio, value at risk, conditional value at risk.

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INTRODUCTION

Regularly, Investors are vulnerable to a range of risks, and without a thorough and sound analysis of these risks, investments may bring about losses. Therefore, effective risk control is a must. At the institutional level, regulatory bodies have commenced placing on restrictions to limit exposure to several types of risks. For example, the 1993 G30 meeting and the 1995 Basel Committee on Banking Supervision set guidelines and recommendations for market risk management. The 1998 Basel Accord stressed credit risk management, and several models were put forward for measuring risk. However, market risk is the most important risk, defined as the possibility of an investor incurring losses due to factors that affect performance. It is also a systematic risk that cannot be threw out through diversification, but can be hedged in other ways. This contrasts with so-called unsystematic risks, which are unique to certain companies and can be get rid of through diversification. The main types of market risks involve interest rate, commodity, and currency risks, among others (Colivicchi et al., 2019: p. 31).

With the spreading of this type of risk in the financial markets, assessing the extent of exposure to market risks has been converted into one of the most important tasks of financial institutions and specialists. This is usually brought about through several models, as the market risks faced by financial institutions during sharp market fluctuations can arouse radical changes in investment values, which influences the performance of companies. On that ground, market risk measures have begun to render an estimate of the worst possible financial loss for an investment during a specific period of time, and for a specific level of reliability (or probability). (Chan et al., 2007: p556).

It is substantial to mention that studies in the literature of financial management and its applications in the recent period have been specific about value and the extent of its exposure to risk, with the emergence of engineering and re-engineering concepts and developments in techniques and methods for estimating the potential risks of investment returns, which are known as VaR models. (Rachev & Khindanova, 2002: 8) In 1994, J.P. Morgan published a risk control methodology known as RiskmetricsTM, which was primarily based on a modern measure of financial risk called VaR, which is one of the approved measures that represents an option for measuring market risks, which regulatory bodies and institutions can use to figure out capital requirements (Chan et al., 2007: p. 556). This methodology was held to be a masterpiece in financial risk management and quickly gained widespread popularity. Over the past few years, VaR has become a fundamental component of market risk management for many financial institutions. It is exploited as an internal risk management tool and has been opted for by the Basel Committee as an international standard for regulatory purposes. VaR is a simple risk measure used by financial institutions to assess their portfolios' exposure to market risk. The main feature of VaR is the aggregation of potential losses that may take place with a certain probability in a specific time period into a single value. It is based on the so-called profit or loss distribution in the model proposed by RiskMetrics (Lamantia et al., 2006: p1).

Accordingly, this study employed 13 time periods, one year for each period, i.e. for (13) investment portfolios including (11) company stocks. This study was partitioned to a group of main paragraphs. The first paragraph contained, as shown, an introduction to the study. The second paragraph included literature reviews. The third paragraph covered the development of hypotheses, while the fourth paragraph comprised the methodology followed in the study. The fifth paragraph paid attention to the results. The sixth paragraph brought about the end of this study under the title "discussions".

LITERATURE REVIEW

Value at Risk

Value at risk (VaR) was not a conventionally used term before the mid-1990s, but its origins date back much further. The mathematics underlying VaR were considerably improved in the context of portfolio theory by Harry Markowitz and others. Several terms were coined before VaR,

especially during the 1990s, including Dollars at Risk (DaR), Income at Risk (IaR), Income at Risk (CaR), Earnings at Risk (EaR), and Value at Risk (VaR). All of these terms were based on what is known as VaR. DaR has been under criticism for not being comprehensive to all businesses. CaR has been found fault with not using capital in their models. LaR and (R) Terms that are not related to the overall risk, especially market risk. Hence, VaR was employed because it encompassed the aforementioned concepts (Glyn, 2002: p24). Even though efforts were initially directed toward designing optimal portfolios, focusing on market risk and the effects of associated changes in these risks is central to how VaR is calculated. VaR is described as a risk measurement method that statistically estimates the maximum potential loss of an investment over a specified period of time at a given confidence level under normal market conditions. VaR speaks for the amount of loss investors would incur over a specified investment period at a confidence level of $\alpha-1$, expressed as a percentage of units or currency (Astuti &Gunarsih, 2021: p. 106). It is therefore one possible measurement method for estimating potential losses when the price of an asset or portfolio declines. A portfolio's VaR symbolizes the maximum amount an investor could lose over a certain period of time with a given probability (Abada et al., 2014: p. 15). The VaR sums up the worst-case loss over a target time horizon at a specified confidence level. More accurately, VaR illustrates the percentage distribution of expected gains and losses over the target time horizon (Ryabtsev &Ryabtsev, 2011: p. 157). The VaR can also be used as a warning to avoid the worst financial risks (Syuhada, 2020: p. 1). Consequently, the VaR was published through the Bank for International Settlements' regulations and adopted as a standard method for assessing market risk. The abbreviated VaR concept depicts the value of risk at a confidence level of α for a period of t days, meaning that losses exceeding the value of risk occur with a probability of $100\% (\alpha - 1)$ within t days (Kozaki & Sato, 2008: p. 1226). It is also exploited as a tool to measure insurance companies' exposure to risk (Chan et al., 2007: p. 556).

It is also among the most routinely used risk management and assessment tools for measuring the risks of investment assets or investment funds in both highly liquid and less liquid financial markets. The VaR calls for the so-called tail risk, or the risk of severe downside that an investor may be exposed to when holding an asset or portfolio for a specific period of time and at certain confidence levels. In simpler terms, the VaR assesses the worst possible loss from an investment option. Due to the ease of use and simplicity of the VaR model, various market participants (such as policymakers, financial intermediaries, and portfolio or fund managers) have used this tool to deal with adverse market movements and market shocks. Unlike traditional risk measures, the VaR bestows an aggregate view of portfolio risk, taking into account leverage and correlations between assets. Consequently, it is a truly forward-looking risk measure (Ryabtsev, 2011: p. 158). Under the VaR strategy, the insured portfolio is constructed and often rebalanced, such that the portfolio level exceeds a minimum threshold at a certain confidence level at each time step (Alipour & Bastani, 2023: p3). The VaR has thus turned out to be a standard for risk reporting because it captures an important aspect of risk, namely market risk (Di Clemente & Romano, 2005: p30). The VaR has turned widely used theoretically and quantitatively as a potential method for assessing future risk (Zhang et al., 2018: p5280).

Nevertheless, VaR has faced many criticisms, including its unrealistic assumptions regarding linear and normal distributions, its sensitivity to the estimation and holding period, and its inadequacy during crises, especially when there is variation in correlations between assets. The latter point is of particular importance, as it is noted that the impact of correlations is much greater than in periods of market stability (Cho et al., 2022: p. 197). Besides, some VaR models take no notice of some types of correlations, or some of them are illogical, or some correlations do not accurately reflect the stock markets. Therefore, proposing a model that accurately reflects changes in the stock markets is of paramount importance (Zhang et al., 2018: p. 5281). Furthermore, Gao & Liu (2009) identify three criticisms: first, the nature of distributions, which does not lead to a comprehensive measure of risk; second, the difficulty with discrete data, especially within investment portfolios; and finally, VaR does not furnish a complete picture of the magnitude of risk, but rather a percentage (Gao & Liu, 2009: p. 246). Most risk models particularize on short-term risks, neglecting the long-term, which played a role in the financial crisis. Short-term risk forecasting is usually a direct result of manipulating historical returns as inputs, which cannot be converted into long-term outputs. Short-term forecasting is, in effect, a late warning for most illiquid assets that cannot be liquidated in the short term (Molino & Sala, 2021: p. 1191).

The bright side, particularly in estimating the expected probability density function (VaR) and even the expected probability density function (CvaR), is that once the expected probability density function relevant to the data used is known, the expected probability density function for the stock or portfolio can be estimated. Estimating the probability density function using historical data is simply not possible without assuming that the historical data used in the estimation represents a good indicator of the future (Molino & Sala, 2021: p1192). If the cumulative density function is known, the VaR is simply its p-percentage. Since the cumulative density function is unknown in practice, value-of-risk studies focus primarily on estimating the cumulative density function, particularly its tail behavior. In general, as for calculating the VaR (Colivicchi et al., 2019: p32) includes the following elements:

1. Probability
2. Time horizon
3. Cumulative distribution function, symbolized by $F(x)$, or its quantity.

On that account, unexpected losses are measured using VaR, which assesses the maximum loss that can be recorded over a specified period of time for a given confidence interval. Generally, this value represents a multiple (k) of the standard deviation:

$$\text{VaR} = k \times s$$

Within a probability distribution, the VaR value represents the difference between the expected loss and the maximum loss that can be received with a given confidence level. The α value represents a very low value (e.g., 1%, 5%, 10%). These coefficients measure the probability of losses greater than the value of the risk (Beltrame et al., 2014: p. 54). Value-at-Risk approaches

are further classified into two main approaches: parametric and non-parametric, as well as semi-parametric approaches, as follows: (Abada et al., 2014: p. 15).

Parametric Framework

The parametric framework hypothesizes that the data follow a known and described probability distribution. Market risk management typically refers to a normal distribution. In this case, the relevant change is the portfolio loss, not the market return. The return is characterized by symmetric behavior and often centers around zero. However, the distribution of losses is not completely symmetrical, with a non-zero mean. Hence, alternatives to the normal distribution, such as the beta distribution, are necessary. Given the distribution parameters, it is possible to obtain the percentage at the desired confidence level. Therefore, the VaR is calculated as the difference between the expected loss and the maximum loss for a given confidence level, according to the following formula:

$$\text{VaR} = \text{loss max} - \text{EL}$$

Even though the parametric approach is simple, it is often limited by the distribution behavior such as thick tails, skewness, kurtosis, etc. Moreover, before using the parametric distribution, it is necessary to conduct a series of tests to verify whether the parametric model represents a valid support for interpreting the behavioral effect of the data. This is a very long and expensive process (Beltrame et al., 2014: p55-56). This approach includes two models: the variance/covariance model and the quadratic approximations model. VaR is calculated according to the normal distribution of returns σ as follows:

$$\text{VaR}_\alpha = -\sigma F_{-1-\alpha}$$

Where (σ) stands for the variance value over the period, and ($F_{-1-\alpha}$) represents the table value according to a specified probability K . One of the most prominent parametric models is the GARCH model, developed by Bollerslev (1986) from the autoregressive moving average (ARMA) process. However, this process cannot symbolize asymmetric variance patterns (Chang et al., 2011: p. 266).

GARCH can be used for stock prices, financial indices, and foreign exchange rates, and there is a growing literature on its applications in asset pricing and risk management. (Chan et al., 2007: p. 556) It is a widely used model for modeling and forecasting the daily volatility of financial returns. Its estimation and forecasting results in the presence of extreme values have been documented in previous studies. According to the GARCH model, the evolution of the squared variance is illustrated by the following equation (Trucíos et al., 2020: p. 2583):

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

Due to the extreme values experienced by some returns, the following equation was adopted: ω is a constant to ensure Fisher consistency:

$$\sigma^2_t = \omega + \alpha y_c r_c \left(\frac{r^2_{t-1}}{\alpha^2_{t-1}} \right) + \alpha^2_{t-1} + \beta^2_{t-1}$$

The GARCH and other approaches are one of the procedures deployed in applying different VaR models. Many experts have adopted various models to measure asymmetric power VaR, making specific assumptions about distributions. They have concluded that using a combination of different methods produces a VaR measure that provides a more accurate prediction of future losses for different investment opportunities. Others have used VaR measures that combine GARCH models with different return distributions, and they have concluded that the model assuming “t” distribution performs better than other measures. The effectiveness of different VaR measures has also been tested using GARCH models, and it has been shown that the VaR model outperforms using exponential GARCH. Many subsequent studies have been conducted to investigate different GARCH models in an effort to identify the most effective VaR model for risk management. Some have attempted to present a generalized VaR model, called Lambda VaR, and demonstrate its competitiveness using a different testing procedure (Ryu et al., 2022: p3-4).

Non-Parametric Framework

The parametric approach is closely identical to the non-parametric approach. They both are after constructing an empirical probability distribution and calculate VaR according to percentage logic, or scale the distribution to a loss level where the frequency associated with the largest losses is less than or at most equal to α . The essence of this approach is to allow the data to express itself as much as possible, as VaR is measured without making strong assumptions about the distribution of returns. All non-parametric approaches are based on the fundamental assumption that the near future will be similar enough to the recent past that data from the recent past can be used to predict near-future risks. This is a simulation approach, derived through the application of simulation steps, and includes two models: the Historical Simulation Model and the Monte Carlo Simulation Model.

Historical Simulation Model

This traditional method uses samples from historical data to predict the value of risk (VaR). One form of HS is filtered historical simulation (FHS), which calculates sample ratios from filtered residuals using a barometric model such as AR-GARCH (Peng et al., 2023: p5). The historical simulation method creates a probability distribution based on historical information. The basic theory is that historical data can be easily interpreted even in the future. Because outcomes are predictable, this requires large data sets, which are not always available. To overcome the problem of lack of data, new data can be generated, and specific formulas can reproduce what appear to be random phenomena. The numbers generated by mathematical formulas are called random number generators (RNGs) because they are not completely random. RNGs simulate, at this stage, all risk factors are involved and produce a different effect with each simulation. If the number of simulations is large, the likely outcomes will be precisely determined, providing results that are often difficult to achieve using traditional methods. Once a distribution of probability values that

can be assumed to be a loss is obtained, the outcome can then be regularized according to size, excluding extreme losses that occur with a frequency less than or equal to α , with the associated maximum loss value at the chosen confidence level. More recently, limitations of VaR, such as the impossibility of predicting maximum future losses based on historical distributions, have been overcome through conditional VaR (expected shortfall), which takes into account the average losses that can be verified beyond a confidence interval (Beltrame et al. al., 2014: p56). VaR is measured according to the following formula (Rydell, 2013:p 4):

$$\text{VaR}_\alpha(X) = \inf\{x \in \mathbb{R} : P(X > x) \leq (1 - \alpha)\}$$

Monte Carlo Simulation Model

Monte Carlo simulation is another non-parametric model and one of the most popular with VaR. Nevertheless, it is also the most complicated to implement and generates a large number of scenarios based on assumptions. However, the Monte Carlo (MC) simulation method faces the problem of ignoring the widely observed heteroskedasticity in real markets, which results in heteroskedastic return distributions, which can underestimate risk. MC assumes that returns follow a specific time series model, such as the GARCH model, and evaluates VaR numerically using Monte Carlo simulation. Being a numerical method, MC does not impose many constraints on models. However, the heavy load of simulation is analyzing the impact of changing portfolio weights, which may require several simulations. VC also suffers from the disadvantage of underestimating risk, while MC requires significant computational resources and faces analytical difficulties (Kozaki & Sato, 2008: p. 1226). The variance–covariance VC method evaluates VaR assuming normally distributed returns. Its name comes from the fact that the variance–covariance VC matrix of assets plays an important role in the method. Based on portfolio theory, it permits for simple analysis and calculations, which facilitated variance–covariance to be widely used early in practice.

Semi-Parametric Framework

The GJR-GARCH-FHS model is one of the most remarkable semi-parametric approaches. It is parametric in figuring out the variance of the GJR-GARCH part and non-parametric in determining the FHS part. It is empirically known that index returns exhibit random volatility, negative skewness, and excessive kurtosis. It is also well established that volatility groups for periods of high and low volatility revert to their mean and respond more strongly to negative events than to positive events. One of the characteristics of the GJR-GARCH-FHS is that it is able to accurately predict variances over short and long time horizons (Molino & Sala, 2021: p1191). The CVaR is a risk measure that can be calculated parametrically, non-parametrically, or semi-parametrically. CVaR, also apparent as expected default, or ES, has gained significant attention in the financial risk management literature as an effective risk indicator. CVaR at level α represents the conditional expectation of losses in the top 100% ($\alpha - 1$), and is envisioned to be a better risk indicator than Value at Risk (VaR).

While the use of VaR is partly accounted for the 2007-2008 financial crisis, CVaR is more attractive than VaR because it takes into account the contribution of very rare but very large losses. CVaR is also a robust measure of risk. CVaR is robust, but several observations are required to accurately estimate it because it is a tail statistic. However, historical data older than 5 years may be irrelevant due to the non-stationarity of the return distribution. Therefore, from a practical perspective, data-driven portfolio optimization involves estimated statistics that are subject to potentially significant estimation errors. Such observations have been made in the context of mean-variance optimization (Lim et al., 2011: p. 164). Additionally, the conditional value-at-risk (CovAr) measure has emerged as one of the most important metrics that takes into account the level of extreme tail dependence among financial institutions at specific significance levels (P 6-10) (Tobias & Brunnermeier, 2016). $\Pr(X_i \leq \text{VaR}_i q) = q\%$

Where (X) is the series of crises of the returns of financial institution (i), and since VaR is positive, a rise in this value signifies an increase in the financial institution's exposure to risk.

Portfolio Model and Selection

Investment portfolio managers encounter a set of challenges, starting with constructing an investment portfolio and adjusting its weightings according to the return and risk offered by each asset. This is conducted by the investment portfolio manager under an advisory, guidance, or portfolio management contract (Lee & Sohn 2023:44). Besides, modern portfolio theory seeks to allocate assets to maximize the expected risk premium per unit of risk, within the framework of mean and variance. Focusing on standard deviation means that investors weigh the probability of negative returns equally with positive returns, which is inconsistent with the reality that returns are typically abnormal, skewed, and have a thick tail. Furthermore, there is ample evidence indicating that losses and gains are often treated asymmetrically. In response to these problems, optimal portfolio selection models have been developed to maximize expected returns while taking into account a risk constraint rather than a standard deviation. These models are more befitted to the individual perception of risk and the constraints encountered by management (Scheller & Auer,2018:p2009).

It is significant to mention that the problem of maximizing the total expected utility of consumption or wealth over a given time period [0T,] is often formulated at time $t = 0$, where the investor has an initial wealth of X_0 . The problem is how to allocate funds between investment and/or consumption over a given time period (Yiu, 2004: p1219). Hence, the portfolio allocation problem is defined as allocating assets and choosing the amount to borrow or lend in such a way as to achieve the maximum expected level of final wealth. Specialists introduce a constraint into the basic portfolio selection problem that focuses on the level of the desired dollar value of risk (VaR). Therefore, the constraint takes the following form (Scheller & Auer, 2018: p2009):

$$P\{w(0) - w(T,p) \geq \text{VaR}\} \leq (1 - \alpha)$$

Where $P_0(0)$ stands for the regular probability conditional on the information available at time 0 for portfolio p , and thus the constraint states that the probability of a loss exceeding the VaR level set by the investor should not be greater than one minus the confidence level α , for the optimal portfolio, as it maintains the condition of the previous equation, which will directly reflect investors' risk aversion because it is related to both the VaR level and the associated confidence level (Scheller & Auer, 2018: p2009).

Building an Ideal Investment Portfolio

The basis of portfolio construction is concerned with rational financial decisions in terms of balancing return and risk. The essence of portfolio theory revolves around the impact of considered diversification. An investor can choose his optimal portfolio after identifying his indifference curves in the form of the efficient set and then selecting the portfolio located in the northwest corner of the investor's indifference curves, as it lies on the efficient frontier and within the feasible region of available portfolios (Alexander et al., 2001:149). Relying on the efficient frontier for investment portfolios, whether composed of two or more assets, the optimal investment portfolio is one of the possible portfolios composed of two assets along a straight line or more than two assets along a curve connecting two assets (Brigham & Ehrhardt, 2014:983). In line with the framework of many empirical studies and in accordance with the previous constraint that gives permission to the establishment of a rule for selecting an optimal investment portfolio, and since the expected return on an investment portfolio is $r(p)$ for portfolio p in period T , the expected final wealth from investing in p is:

$$E_0(W(T, p)) = (W(0) + B)(1 + r(p)) - B(1 + r_f)$$

Developing Hypotheses

Investors look for investment returns commensurate with the investor's acceptable risk profile. Building and diversifying an investment portfolio is one way to reduce risk. Conventionally, investment risk is measured through standard deviation or variance. With the adoption of more appropriate models and approaches, CVaR has settled into a more effective method for capturing risk after VaR, which is used as a constraint within an investment portfolio. Therefore, the study developed a basic hypothesis centered around the following question: Is CVaR a reliable measure of risk, and does it compose a constraint for constructing the optimal investment portfolio?

METHODOLOGY

Sample and Measurement

In this study, there was a reliance on returns calculated using the closing prices of a sample of companies listed on the Iraq Stock Exchange for the period 2012-2024. At various confidence levels of 99%, 95%, and 90%, to calculate both VaR and CVaR, in addition to the main indicators (realized profits and losses) for each stock within the investment portfolio and according to the periods, the period was divided into thirteen periods and investment portfolios were built with a value of 10 million dinars for each portfolio for a period of time, from 11 shares of the companies'

shares with equal weights and (12) observations with the aim of arriving at results that prove or deny the main hypothesis of the study. Appendix (1) displays the descriptive statistics for the profits and losses of the name of the investment portfolios during each holding period multiplied by the value of the investment portfolio in preparation for entering them into the calculation of VaR. Table (1) brings to light some descriptive statistics for both VaR and CVaR for the shares of the (11) companies in the study sample and for the (13) periods extracted from Appendix (1) which encompasses the values of profits and losses for the companies' shares.

Table (1): Descriptive Statistics for both VaR and CVaR

		VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
Confidence Level 90%	MAX	-2,834,733	-28,347,334,768	-5,144,206	-51,442,059,248
	MIN	-11,762,234	-117,622,338,691	-	-
	AVERAGE	-7,138,710	-71,387,099,347	16,790,196	167,901,959,515
	SD	3,032,628	30,326,281,364	-9,557,120	-95,571,197,651
Confidence Level 95%	MAX	-4,926,721	-49,267,208,075	-6,447,384	-64,473,844,233
	MIN	-16,313,150	-163,131,501,187	-	-
	AVERAGE	-9,338,893	-93,388,927,445	21,560,654	215,606,542,791
	SD	3,717,504	37,175,037,615	-	-
Confidence Level 99%	MAX	-6,270,121	-62,701,206,288	-6,447,384	-64,473,844,233
	MIN	-20,511,153	-205,111,534,471	21,560,654	215,606,542,791
	AVERAGE	-11,259,291	-112,592,905,255	-	-
	SD	4,591,061	45,910,607,928	11,739,390	-117,393,899,707

The table is prepared by the researcher based on EXCEL results.

RESULTS

In this part, an analysis of both VaR and CVaR will be carried out according to the three confidence levels: 90%, 95%, and 99%, as follows:

Value at Risk VaR

As far as historical simulations of investment portfolio returns are concerned, it turned out to be clear, through the data in Table (2), which displays the 13 periods under which the investment portfolios for the 11 companies' stocks were constructed, that the largest value at risk achieved during these periods was in the third period, at -11,762 thousand, meaning the maximum loss

incurred by the investment portfolio at a 90% confidence level. Therefore, there is a 10% probability that the portfolio will incur losses greater than -11,762 thousand. This is because when an amount of 10 million dinars is invested within the third period (portfolio), this is the maximum loss that will be incurred within this probability. At a confidence level of 95% and a probability of 5%, and through the data in Table (3), we also find that the third portfolio achieved the largest VaR, which amounted to -16,313 thousand, and thus there is a probability of 5% that this portfolio will achieve losses greater than this value. Table (4) also showed the VaR at a confidence level of 99% and a probability of 1% that the third portfolio within the periods achieved the largest VaR, which amounted to -20,511 thousand, and therefore there is only a probability of 1% that this portfolio will bring about losses greater than -20,511 thousand.

The lowest loss that the investment portfolios will encounter during the 13th period will be in the second period, as the value at risk reached -2,835 thousand, meaning the lowest losses that the investment portfolios will face at a 90% confidence level. Therefore, there is a 10% probability that this portfolio will incur a loss greater than -2,835 thousand. Therefore, investing 10 million dinars in the second portfolio will incur this lowest loss among the portfolios (periods), according to the data in the table. At a 95% confidence level and a 5% probability, we also find that the second portfolio achieved a VaR of -4,927 thousand. Therefore, there is only a 5% probability that the second portfolio will incur losses exceeding the minimum of -4,927 thousand, according to the data in Table (3). The second portfolio also achieved the lowest VaR of -6,270 thousand at a 99% confidence level. Consequently, there is only a 1% probability that this portfolio will incur losses greater than the minimum of -6,270 thousand, as referred to in the results of Table (4).

Conditional Value at Risk (CVaR)

Disclosing the data in Table (2) and the section related to the results of calculating the Conditional Value at Risk (CVaR), in this study there was no limit to the loss ceiling as is the case with Value at Risk (VaR). Rather, we calculate the actual loss of investment portfolios when the loss exceeds the VaR limit. The results are presented according to CVaR, as well as based on historical simulations and various confidence levels. At a 90% confidence level, the third portfolio achieved the largest actual loss according to CVaR, amounting to -16,790 thousand. Therefore, the expected losses within this portfolio are very high in the worst cases, meaning that risks are high in crises and losses are large in bad market conditions. There is only a 10% probability that the losses of the third portfolio will exceed this loss. As is the case with VaR, the third investment portfolio also achieved the largest CVaR at a 95% confidence level of 21,561 thousand, meaning that there is only a 5% probability that the third portfolio will achieve a CVaR greater than 21,561 thousand, as shown in Table (3), and also in Table (4), and at a 99% confidence level, the third portfolio achieved the largest CVaR of 21,561 thousand. What is remarkable here, after examining the calculation process, is that some values, specifically at a 95% and 99% confidence level, are equal, and there are several causes, including the distribution having a small number of maximum values, or that the maximum loss shows up before reaching a 95% confidence level, in addition to the

absence of additional losses after a certain limit, or that the distribution in the tail (bell) of CVaR is short.

The lowest CVaR within a 90% confidence level realized during the 13 periods was also within the second period, reaching -5,144 thousand, as made clear in Table (2). Hence, there is only a 10% probability that losses will exceed the minimum limit within this portfolio. That is, the second portfolio will incur a loss estimated at -5,144 thousand when the investment portfolio value is 10 million dinars. There is only a 10% probability that this value will be exceeded. This represents that the risks are relatively low even in times of crises. Therefore, the portfolio is safer and more stable compared to the remaining portfolios. As for the 95% and 99% confidence levels, the situation and reasons are the same as for the highest value. The second portfolio achieved the lowest value, reaching -6,447 thousand. Therefore, there is a 5% and 1% probability, respectively, that the second portfolio will incur losses exceeding the minimum limit, as shown in Tables (4) and (3). The tables also included the various confidence levels, VaR AMOUNTS and CVaR AMOUNTS, whose rankings do not differ within investment portfolios, as both VaR and CVaR stand for a percentage of the capital invested in the investment portfolio, i.e., the value of the portfolio.

Table (2): Values at Risk for the Periods Group, 90% Confidence Level

period / V	VaR	VaR AMOUNTS	CVaR	CVaR AMOUNTS
period 1	-8,599,275	-85,992,748,154	-14,594,355	-145,943,546,632
period 2	-2,834,733	-28,347,334,768	-5,144,206	-51,442,059,248
period 3	-11,762,234	-117,622,338,691	-16,790,196	-167,901,959,515
period 4	-10,495,935	-104,959,351,307	-11,688,310	-116,883,099,152
period 5	-8,288,636	-82,886,356,131	-10,311,714	-103,117,136,596
period 6	-9,810,521	-98,105,205,057	-13,282,801	-132,828,006,159
period 7	-9,881,238	-98,812,376,384	-11,578,127	-115,781,268,802
period 8	-6,193,664	-61,936,635,146	-7,820,281	-78,202,813,676
period 9	-3,414,897	-34,148,971,675	-5,694,524	-56,945,235,000
period 10	-4,675,645	-46,756,450,769	-5,641,640	-56,416,399,028
period 11	-4,981,577	-49,815,766,248	-6,847,534	-68,475,344,024
period 12	-3,414,080	-34,140,799,871	-6,042,714	-60,427,138,554
period 13	-8,450,796	-84,507,957,308	-8,806,156	-88,061,562,075

The table was prepared by the researcher based on EXCEL results.

The following figure shows the degree of agreement or discrepancy between VaR and CVaR, at a 90% confidence level

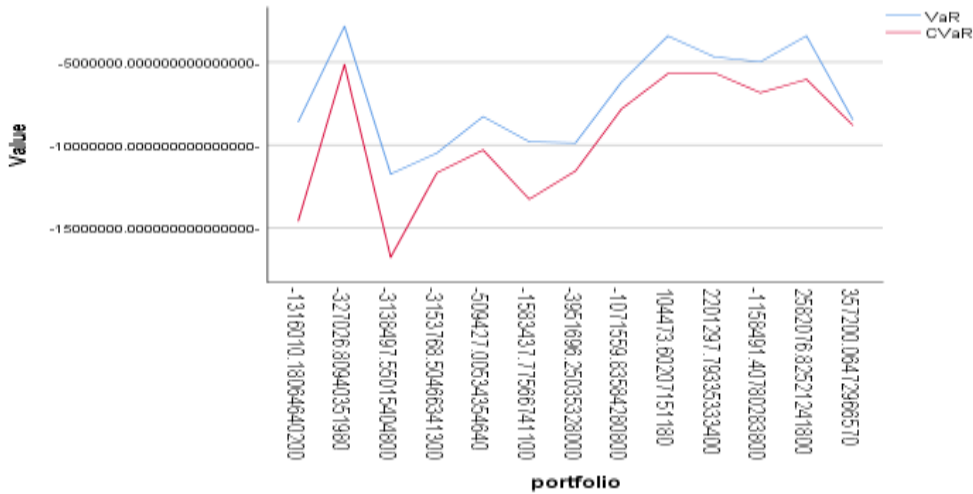


Figure prepared by the researcher based on the values data and using the SPSS statistical program.

Table (3): Values at Risk for the Period Group at the 95% Confidence Level.

period	VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
period 1	-14,064,451	-140,644,512,425	-19,893,389	-198,933,888,704
period 2	-4,926,721	-49,267,208,075	-7,319,057	-73,190,570,984
period 3	-16,313,150	-163,131,501,187	-21,560,654	-215,606,542,792
period 4	-11,592,661	-115,926,605,161	-12,644,804	-126,448,039,070
period 5	-10,124,362	-101,243,618,240	-12,185,232	-121,852,320,153
period 6	-12,994,743	-129,947,434,242	-16,163,373	-161,633,725,329
period 7	-11,435,525	-114,355,245,774	-13,004,150	-130,041,499,083
period 8	-7,674,830	-76,748,300,599	-9,274,794	-92,747,944,444
period 9	-5,481,493	-54,814,929,713	-7,824,830	-78,248,298,870
period 10	-5,561,065	-55,610,654,507	-6,447,384	-64,473,844,233
period 11	-6,671,317	-66,713,174,803	-8,609,704	-86,097,036,227
period 12	-5,791,144	-57,911,435,635	-8,558,417	-85,584,167,751
period 13	-8,774,144	-87,741,436,426	-9,126,282	-91,262,818,557

The table was prepared by the researcher based on Excel results.

The following figure makes known degree of congruence or variance between VaR and CVaR at a 95% confidence level.

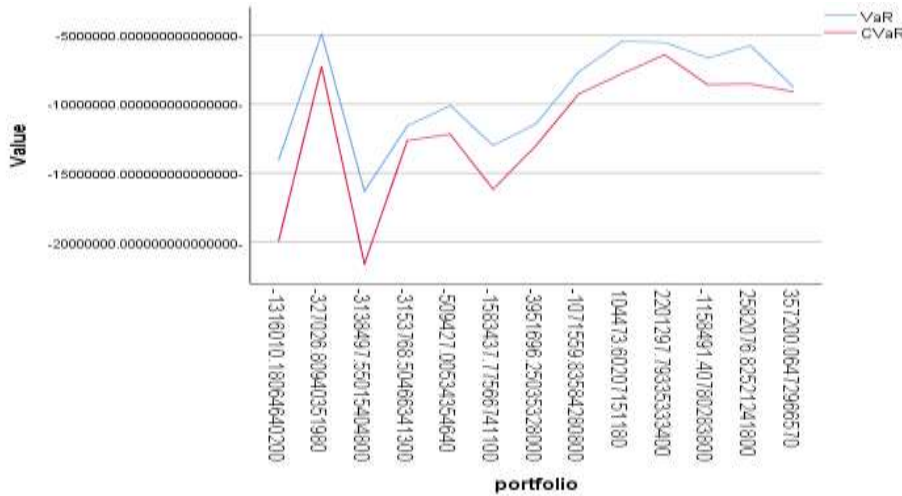


Figure was prepared by the researcher based on the value data using the SPSS statistical program.

Table (4): Values at risk for the Periods Group at the 99% confidence level

period	VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
period 1	-18,727,601	-187,276,013,449	-19,893,389	-198,933,888,704
period 2	-6,840,590	-68,405,898,402	-7,319,057	-73,190,570,984
period 3	-20,511,153	-205,111,534,471	-21,560,654	-215,606,542,792
period 4	-12,434,375	-124,343,752,289	-12,644,804	-126,448,039,070
period 5	-11,773,058	-117,730,579,771	-12,185,232	-121,852,320,153
period 6	-15,529,647	-155,296,467,112	-16,163,373	-161,633,725,329
period 7	-12,690,425	-126,904,248,421	-13,004,150	-130,041,499,083
period 8	-8,954,802	-89,548,015,675	-9,274,794	-92,747,944,444
period 9	-7,356,163	-73,561,625,038	-7,824,830	-78,248,298,870
period 10	-6,270,121	-62,701,206,288	-6,447,384	-64,473,844,233
period 11	-8,222,026	-82,220,263,942	-8,609,704	-86,097,036,227
period 12	-8,004,962	-80,049,621,328	-8,558,417	-85,584,167,751
period 13	-9,055,854	-90,558,542,131	-9,126,282	-91,262,818,557

Table was prepared by the researcher based on EXCEL results.

The following figure discloses the degree of agreement or discrepancy between VaR and CVaR at a 99% confidence level.

eriod	V/Stock	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10	Stock11
1	MAX	1,612,681	914,923	1,264,841	884,110	820,132	2,330,93	1,392,62	636,257	1,119,17	914,342	1,435,482
	MIN	-5,108,256	0	2,461,331	-806,889	-336,301	1,775,48	1,064,83	3,677,248	1,521,91	1,112,25	-4,752,347
	AVERAGE	-355,432	-316,480	-7,610	-39,690	0	181,878	-52,100	-228,241	-9,862	51,036	-549,371
2	MAX	1,656,047	833,816	1,442,496	1,449,340	859,424	833,816	941,872	1,335,314	1,758,90	1,397,61	1,230,601
	MIN	-2,310,632	5	1,885,912	-702,043	3,051,567	-843,811	1,283,81	1,185,597	1,887,94	1,033,78	-2,231,436
	AVERAGE	119,441	-366,076	40,633	263,771	-239,683	-1,242	-179,803	52,879	-19,295	-27,158	61,525
3	MAX	2,894,132	0	1,407,496	5,358,269	2,316,320	1,289,70	2,180,02	1,076,307	3,028,11	1,139,44	1,231,327
	MIN	-3,990,753	4	2,605,311	4,744,580	2,261,242	1,053,60	1,209,52	5,978,370	1,636,29	-943,107	-2,309,056
	AVERAGE	-629,917	-418,933	-185,430	-539,893	-282,241	-96	-3,506	-665,533	70,124	21,341	-214,076
4	MAX	2,100,715	2	1,631,953	2,052,631	1,473,247	540,672	1,793,40	2,651,078	1,970,17	1,127,95	2,014,217
	MIN	-3,101,549	1	2,363,888	-976,090	2,363,888	-565,704	2,570,45	3,545,450	2,541,75	2,181,56	-1,973,594
	AVERAGE	-491,857	-533,581	-451,175	-14,537	-387,928	-39,464	-428,184	-261,093	-373,434	-295,022	-232,814
5	MAX	3,610,133	3	1,372,011	1,216,969	1,112,256	0	1,941,56	1,251,631	1,872,11	1,743,53	1,119,179
	MIN	-1,112,256	7	1,625,189	-930,904	1,582,240	-540,672	2,058,52	2,411,621	3,038,82	2,682,64	-1,905,183
	AVERAGE	146,742	-177,145	-130,985	98,275	91,412	-44,626	-213,583	-151,334	-254,696	-118,973	-211,228
6	MAX	1,251,631	2	1,823,216	1,053,605	2,559,334	0	3,237,87	2,992,429	3,639,65	971,637	2,876,821
	MIN	-759,859	0	2,177,235	1,521,918	2,047,944	-44,626	1,670,54	3,001,046	1,133,28	1,686,22	-2,876,821
	AVERAGE	-64,082	-243,237	-120,153	-166,034	22,893	-3,433	-112,709	-169,989	85,466	-283,517	-376,101
7	MAX	416,727	8	741,080	1,823,216	953,102	112,996	1,431,00	800,427	5,663,95	909,718	3,361,087
	MIN	-2,929,871	4	1,967,103	2,699,196	-870,114	-224,729	1,431,00	4,054,651	3,591,41	1,251,63	-3,718,267
	AVERAGE	-413,105	-740,456	-522,957	-186,859	-30,295	-264	-213,057	-774,921	-242,493	-333,705	-548,743
8	MAX	1,236,140	8	741,080	1,823,216	953,102	0	909,718	0	7	9	4,855,078
	MIN	-526,437	8	1,431,008	2,231,436	-759,859	1,358,01	1,000,83	1,670,541	2,076,39	2,513,14	-1,941,560
	AVERAGE	23,854	-250,277	-314,593	32,978	-18,874	-436,931	16,349	-261,428	430,972	-53,645	-16,133
9	MAX	1,372,011	1	741,080	531,098	476,280	0	2,513,14	1,053,605	3,170,96	1,743,53	2,513,144
	MIN	-1,112,256	7	-741,080	1,198,012	-930,904	1,076,30	1,643,03	1,053,605	2,199,38	-833,816	-1,957,446
	AVERAGE	-34,802	83,464	-24,199	-121,803	-35,645	-220,492	119,835	120,138	349,240	-127,467	239,047

10	MAX	2,336,149	6,190,39 2	1,941,560	512,933	3,839,589	408,220	2,787,13 4	4,595,323	2,876,82 1	1,431,00 8	5,260,931	
	MIN	-1,100,009	- 1,670,54 1	-	-702,043	2,876,821	-512,933	-723,207	1,335,314	4,480,24 7	1,053,60 5	-500,104	
	AVERAGE	-61,128	251,385	51,210	-94,928	261,062	-56,417	356,899	402,184	250,246	194,582	726,970	
								2,744,36 8	723,207	606,246	2,901,54 3	714,590	1,808,189
11	MAX	870,114	251,385	1,823,216	1,177,830	261,062							
	MIN	-779,615	2,231,43 6	-645,385	-674,413	-582,689	3,877,65 5	1,397,61 9	-	1,335,314	2,513,14 4	-984,401	-1,836,647
	AVERAGE	-46,292	-328,344	53,584	142,935	-145,389	-537,530	-45,862	-243,428	-66,676	-38,104	275,345	
12	MAX	3,593,740	1,541,50 7	4,519,851	3,029,495	2,744,368	2,231,43 6	2,947,99 5	1,670,541	3,469,98 6	5,245,24 5	3,453,112	
	MIN	-1,022,788	1,541,50 7	-	-855,222	1,315,764	1,431,00 8	1,124,78 0	-953,102	1,823,21 6	3,237,87 1	-1,743,534	
	AVERAGE	601,000	77,459	94,724	637,540	269,311	-333,264	34,003	-137,303	604,549	327,066	742,682	
13	MAX	3,400,823	1,177,83 0	8,803,587	2,496,547	2,006,707	2,231,43 6	1,053,60 5	1,335,314	2,246,46 2	3,528,21 4	3,199,429	
	MIN	-1,201,443	- 1,335,31 4	-	-	-	2,231,43 6	2,231,43 6	-	1,376,21 4	1,957,44 6	-3,101,549	
	AVERAGE	93,438	-96,758	694,839	315,676	192,365	-227,454	-256,209	-322,458	380,246	-90,443	204,668	

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